Profile Drag from Laser-Doppler Velocimeter Measurement

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Nomenclature

c = chord length
F_B = force on the solid body in the flow = $F_{vis} + F_\Gamma + F_{buoy}$
F_Γ = force produced by circulation (lift + induced drag)
F_{buoy} = buoyancy
F^* = force in the hypothetical flow = $F_\Gamma^* + F_{source}^*$
F_Γ^* = force due to circulation in the hypothetical flow
F_{source}^* = force on the sources
g = acceleration of gravity
G = potential of gravity
n = normal unit vector
p = static pressure
Re = Reynolds number = $U_\infty c/\nu$
S = control surface
Sw = a part of control surface in the wake
U_∞ = inflow velocity
V = control volume
v = velocity vector
v^* = hypothetical velocity
$x\ y\ z$ = cartesian coordinates
α = angle of attack
ν = kinematic viscosity
$\xi\ \eta\ \zeta$ = transformed coordinates
ρ = fluid density
τ = shear stress
ω = vorticity
ω_Γ = vorticity of trailing vortex
ω_B = vorticity of viscous wake

Introduction

WHEN evaluating the performance of a hydrofoil or airfoil drag characteristics are as important as lift characteristics The drag is usually divided into induced drag and profile drag While the former is associated with the lift distribution and is often assessed analytically the latter has to rely on the measurements Betz[1] derived a convenient formula to calculate the profile drag from wake survey data The formula was given in terms of pressures since wake surveys were then conducted by means of pressure tubes

During the past two decades the rapid development of laser Doppler velocimetry (LDV) has made it possible to measure flow velocities directly in many laboratory test conditions [2 3] Some of the advantages of LDV over the other methods such as Pitot tube or hot wire are as have been discussed in many publications nonintrusive probe, high frequency response high spatial resolution and no need for calibration There have been a number of velocity measurements made by LDV in various flow conditions including those of hydrofoils and airfoils [4 7] Flow surveys around airfoils have been extensively done by Orloff [8 9] and lift and drag distributions were discussed To the author s knowledge however an assessment of the profile drag from the velocity data has not yet been attempted Since the velocities are relatively easily obtained by LDV it may be beneficial to establish a method of predicting the profile drag based on the measured velocity

In this paper using the momentum theorem and the hypothetical flow'' introduced by Betz a prediction formula for the profile drag is derived in terms of velocities The formula is applied to several examples and the results are discussed

Derivation of the Formula

For a solid body (hydrofoil/airfoil) steadily moving in a uniform flow of incompressible fluid, the force exerted by the flow is given by the momentum theorem

$$F_B = F_\Gamma + F_{vis} + F_{buoy} = -\iint_S (\rho v v_n + p n)\,dS - \iint_{S_w} \tau\,dS + \iiint_V \rho g\,dV \tag{1}$$

In order to utilize the measured velocity data, a simplification is desired This can be done by introducing the hypothetical flow,'' which was proposed and used by Betz [1] The requirements primarily imposed on the hypothetical flow are 1) that it be inviscid 2) that on S the static pressure be identical to that of the real flow, and 3) that on S the velocity be the same as that of the real flow except in the wake of the body The hypothetical flow is to be constructed by placing singularities such as sources and vortices, in the flow The forces which act on these singularities are also given by the momentum theorem

$$F^* = F_\Gamma^* + F_{source}^* = -\iint_S (\rho v^* v_n^* + p^* n)\,dS + \iiint_{V^*} \rho g\,dV \tag{2}$$

Subtracting Eq (2) from Eq (1) we have

$$F_B - F^* = F_\Gamma + F_{vis} + F_{buoy} - F_\Gamma^* - F_{source}^* = -\rho \iint_{S_w} (v v_n - v^* v_n^*)\,dS - \iiint_{V_{body}} \rho g\,dV - \iint_{S_w} \tau\,dS \tag{3}$$

where requirements 2 and 3 were used Obviously

$$F_{buoy} = -\iiint_{V_{body}} \rho g\,dV \tag{4}$$

Now to calculate the drag using the aforementioned formula it is desirable that F_Γ^* be equal to F_Γ This can be attained by imposing the same circulation distribution on the hypothetical flow as that in the real flow More details on this will be discussed later For the moment assume that $F_\Gamma^* = F_\Gamma$ Then Eq (3) reduces to

$$F_{vis} = -\rho \iint_{S_w} (v v_n - v^* v_n^*)\,dS - \rho U_\infty \iint_{S_w} (v_n^* - v_n)\,dS - \iint_{S_w} \tau\,dS \tag{5}$$

where Lagally s theorem was used for F_{source}^*

Construction of Hypothetical Flow

In evaluating the profile drag by Eq (5) it becomes necessary to obain the hypothetical velocity in the wake The

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real flow in the wake is described by the Navier Stokes equations which in this case are written as

$$\nabla\left(\frac{1}{2}\,|v|^2+\frac{p}{\rho}+G\right)=v\times\omega+\nu\nabla^2 v \tag{6}$$

The hypothetical velocity on the other hand is supposed to be inviscid and is given by Euler's equations

$$\nabla\left(\frac{1}{2}\,|v^*|^2+\frac{p^*}{\rho}+G\right)=v^*\times\omega^* \tag{7}$$

The vorticity in the real flow consists of two different types or origins; one is called trailing votex and has a direct relationship with the loading distribution over the foil and the other is the one which comes from the boundary layer on the foil to form the viscous wake To take this into account we write

$$\omega=\omega_\Gamma+\omega_B \tag{8}$$

In the hypothetical flow since it is assumed to be inviscid there is only trailing vortex And to satisfy the requirement that the lift be equal to that in the real flow the vorticity has to be equal to that of the trailing vortex Thus

$$\omega^*=\omega_\Gamma \tag{9}$$

If we subtract Eq (6) from Eq (7) taking into account Eqs (8) and (9) we have on control surface S_w

$$\nabla\left(\tfrac{1}{2}\,|v^*|^2-\tfrac{1}{2}\,|v|^2\right)=(v^*-v)\times\omega_\Gamma-v\times\omega_B-\nu\nabla^2 v \tag{10}$$

This is the equation which will give the hypothetical velocity on S_w Solving this equation for v^* however is not simple because of the term $v^*\times\omega_\Gamma$ on the right side To eliminate this term we impose one more requirement, which is

$$v^*-v\,\|\,\omega_\Gamma \tag{11}$$

This requirement is compatible with the one expressed by Eq (9) since irrotational velocity components can be utilized to make the hypothetical velocity satisfy both Eqs (9) and (11) Thus Eq (10) reduces to

$$\nabla\tfrac{1}{2}\,|v^*|^2=\nabla\tfrac{1}{2}\,|v|^2-v\times\omega_B-\nu\nabla^2 v \tag{12}$$

Application to Hydrofoils/Airfoils

A foil is placed in a uniform incoming flow with its span perpendicular to the flow To apply Eq (5) we set up a coordinate system $o\,xyz$ where the origin is placed on the trailing edge at the spanwise location of interest The x-axis is set parallel to the undisturbed inflow; the y axis is perpendicular to the span and the z axis then becomes parallel to the span By definition the x component of the viscous force is the profile drag Taking the x component of Eq (5) we have

$$F_{\text{vis}\,x}=-\rho\int_{y_w}(v_x^2-v_x^{*2})\,\mathrm{d}y-\rho U_\infty\int_{y_w}(v_x^*-v_x)\,\mathrm{d}y \tag{13}$$

To obtain the hypothetical velocity we take the y component of Eq (12) and integrate with respect to y

$$v_x^{*2}+v_y^{*2}+v_z^{*2}=v_x^2+v_y^2+v_z^2-2\int_{y_w}(v_z\omega_{Bx}-v_x\omega_{Bz}+\nu\nabla^2 v_y)\,\mathrm{d}y \tag{14}$$

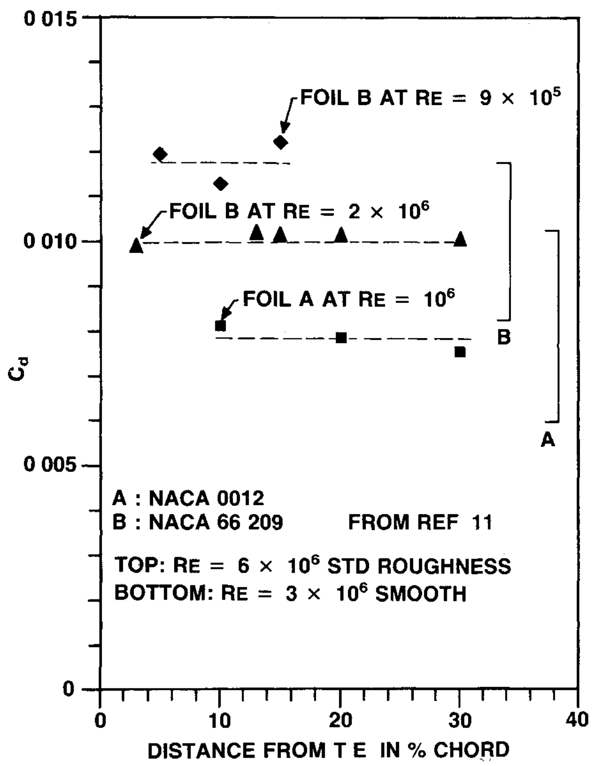

Fig 1 Coefficients of profile drag obtained at different measurement points; Foil A = NACA 0012 basic thickness form $\alpha=0$, Foil B = NACA 66 309, $\alpha=4$ 5 deg

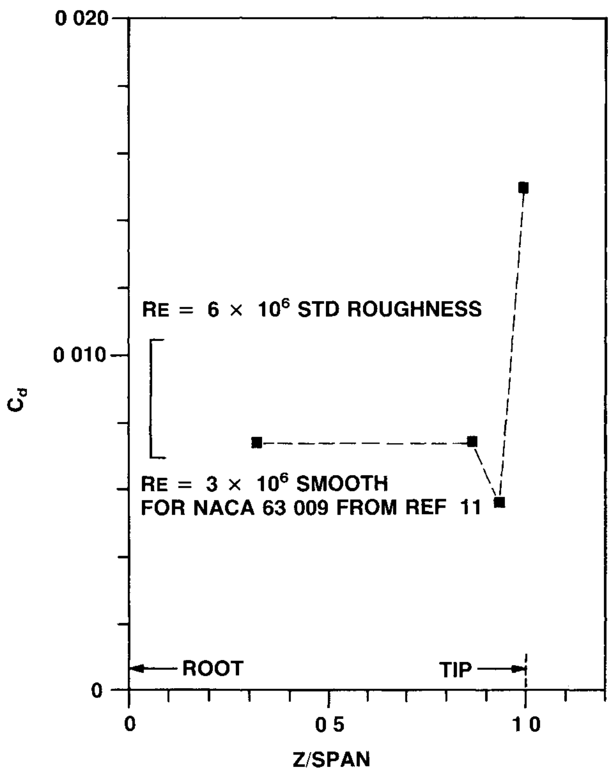

Fig 2 Spanwise distribution of profile drag NACA 63 010 section with 40 deg of sweep back; $\alpha=4$ deg

To solve this we introduce a new coordinate system o − $\xi\eta\zeta$ which is local to the control surface in the wake We set the ξ axis parallel to the trailing vortex there and η axis on the xy plane To find the direction of the local trailing vortex an LDV can be utilized By tracing the velocity jump in the wake the trajectory of the trailing vortex is determined Since the coordinate transformation is by translational and rotational displacements only the absolute value of the velocity is unchanged Then together with requirement (11) Eq (14) reduces to

$$v_\xi^{*2} = v_\xi^2 - 2\int_{y_w} (v_z\omega_{Bx} - v_x\omega_{Bz} + \nu\nabla^2 v_y)\,\mathrm{d}y \qquad 15)$$

For most of the working conditions of a hydrofoil or airfoil it can be assumed that viscous wake vorticity is almost normal to the xy plane Also the curvature of v_y in any direction is considered to be small With these assumptions the equation further reduces to

$$v_\xi^{*2} = v_\xi^2 + 2\int_{y_w} v_x\omega_{Bz}\,\mathrm{d}y$$

The z component of the vorticity for viscous wake ω_{Bz} can be approximated by $\partial v_y/\partial x - \partial v_x/\partial y$ as the z component of the trailing vorticity is considered to be small Thus

$$v_\xi^{*2} = v_\xi^2 + 2\int_{y_w} v_x\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\mathrm{d}y = v_\xi^2 + 2\int_{y_w} v_x\frac{\partial v_y}{\partial x}\,\mathrm{d}y - v_x^2 \qquad (16)$$

v_x^* is now given through the inverse transformation

Results

The values of profile drag computed from the measured velocity distribution by an LDV[10] at various streamwise locations are presented in Fig 1 for two types of foil sections (two dimensional foil) The results for Foil A and Foil B at $Re = 9\times10^5$ have a scatter Since rather old data was used for these two cases in which the use of the formula for the profile drag was not particularly intended there may be some coarseness in the data Foil B at $Re = 2\times10^6$ shows fairly constant value On the same figure the range of values for each case taken from Ref 11 is shown for comparison For Foil B no available data could be found so a similar foil section was chosen instead For Foil A and Foil B at $Re = 2\times10^6$ the computed values compare well with the other source Foil B at $Re = 9\times10^5$ shows values a little higher However since the design lift coefficient for this foil is higher than the one shown for comparison slightly higher values of drag may be expected Figure 2 shows spanwise variation of the profile drag for a finite span swept back foil It is noted that near the tip the value goes down and then climbs up drastically This is due to the highly three dimensional interactions of the boundary layer and tip vortex [10] There profile drag" does not have much meaning The trend shown in the figure is pointed out in Ref 12

Conclusions

A formula has been developed to give the profile drag based on the measured velocity data by laser Doppler velocimetry The formula was applied to some examples and reasonable agreements with the published data were obtained The derivation was based on low turbulence assumption However this assumption may not be valid in some cases It should therefore be extended in order to incorporate the turbulence components In such a case cross terms like $\rho v_x v_y$ will appear in the formula This will suggest a need for simultaneous multicomponent measurements which were not made in the present work

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Engineering Analysis of Drooped Leading-Edge Wings Near Stall

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Introduction

SINCE the late 1970 s low speed wind tunnel experiments[1] and flight tests[2 5] have conclusively demonstrated that wings with a discontinuous leading edge extension and increase in camber (leading edge droop) exhibit a smoothing of

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